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Dynamic Mobility with Single-Wheel Configuration

Abstract

This paper presents a novel concept of mobility that provides dynamic stability and rapid locomotion. The concept, called Gyrover, is a single-wheel, gyroscopically stabilized robot that can be considered as a single wheel actuated by a spinning flywheel attached to a two-link manipulator at the wheel bearing and drive motor. The spinning flywheel acts as a gyroscope to stabilize the robot, and it can be tilted to achieve steering. This configuration conveys significant advantages over multiwheel, statically stable vehicles, including good dynamic stability and insensitivity to attitude disturbances, high maneuverability, low rolling resistance, ability to recover from falls, and amphibious capability. In this paper, the authors present the robot concept, three prototypes of the design, and system implementation. The authors study the robot's nonholonomic constraints and the stabilizing effect of the flywheel on the system through simulation and experiments.

1. Introduction

For decades, researchers have viewed mobile robots largely as quasi-static devices. Numerous robots with four or six (or more) wheels have been developed to maximize mobility on rough terrain (e.g., see Bekker, 1974; Freitag, 1979; Holm, 1970; Kemurdjian et al., 1992; Klarer, 1994). Likewise, legged robots, which may have potentially greater mobility, have been built and demonstrated (e.g., see Krotkov, Simmons, and Whittaker, 1995; Waldron, 1989). Gener-

ally, these robots have featured low center-of-mass placement and broad base of support, along with control and planning schemes designed to keep the center-of-mass gravity vector within the support polygon (e.g., monitoring of slopes, coordination of legs). Many designs have attempted to maximize mobility with large wheels or legs, traction-enhancing tires or feet, multiwheel drive, large body/ground clearance, articulated body configurations, and so on. These robots were often limited by motion-planning constraints and, hence, were designed for low-speed operation, typically 1 kph or less. Dynamic factors have little influence on such systems and, consequently, have been largely ignored.

Traditional research on mobile robotics has placed much emphasis on perception, modeling of the environment, and path planning. Consequently, vehicles have been designed to be compatible with these planning limitations, and the need for high speed has not been evident. Ultimately, as sensing and computation capabilities improve, robots will be limited less by planning and more by dynamic factors. Wheeled robots, capable of dynamic behavior (i.e., high-speed motion on rough terrain) and the exploitation of vehicle dynamics for mobility, represent an exciting and largely unexplored area of research.

A number of researchers have explored the possibilities of using dynamic behavior in various robot linkages, legged locomotors, and other dynamic systems. Examples include Saito, Fukuda, and Arai's (1994) Brachiator and Spong's (1995) robot acrobat, both of which used dynamic swinging motions. Buhler, Koditschek, and Kindlmann (1989) and Schael and Atkeson (1994) studied the dynamics of juggling. McGeer (1989) built a robot that walked down slopes using the periodic swinging motion of the legs, without additional

power or control input. Hodgins and Raibert (1991) built a series of dynamically balanced hopping and running robots that jumped obstacles, climbed steps, and performed flips. Arai and Tachi (1991) and Xu et al. (1998) developed an underactuated robot system using the dynamic coupling between the passive and active joints. Dynamic motion in these systems allowed the emergence of behaviors that would not have been possible quasi-statically.

In parallel with the work on linkage dynamics, other researchers have focused on the dynamics and balance of wheeled robots. Vos and Von Flotow (1990) developed a self-contained unicycle that mimicked the behavior of a human cyclist in maintaining roll and pitch stability. Koshiyama and Yamafuji (1991) developed a statically stable, single-wheel robot with an internal mechanism that could move fore and aft and turn in place; their work emphasized the control of the (noninverted) pendulum carried on the wheel, which used momentum transfer to change direction.

This paper discusses dynamic mobility that is unstable statically and proposes a method using the gyroscopical principle for stabilization of the robot. We address the general background of the concept, mechanism design, prototype development, and dynamic modeling.

2. Dynamic Mobility

Land locomotion can be broadly characterized as quasi-static or dynamic. Quasi-static equilibrium implies that inertial (acceleration-related) effects are small enough to ignore. That is, motions are slow enough that static equilibrium can be assumed. Stability of quasi-static systems depends on keeping the gravity vector through the center of mass within the vehicle's polygon of support determined by the ground contact points of its wheels or feet. Energy input is used predominantly in reacting against static forces. Such systems typically have relatively rigid members and can be controlled on the basis of kinematic considerations.

In dynamic locomotion, inertial forces are significant with respect to gravitational forces. Dynamic effects gain relative importance when speed is high, gravity is weak, and dynamic disturbances (e.g., rough terrain) are high. Significant energy input is required in controlling system momentum and, in some cases, elastic energy storage in the system. As performance limits of mobile robots are expanded, dynamic effects will increasingly come into play. Furthermore, robotic systems that behave dynamically may be able to exploit momentum to enhance mobility, as is clearly demonstrated by numerous human-controlled systems: gymnasts, dancers, and other athletes; stunt bicycles and motorcycles; motorcycles on rough terrain; cars that vault obstacles from ramps; and so on.

It is paradoxical that those factors that produce static stability may contradict dynamic stability. For example, a four-

wheel vehicle that is very low and wide has a broad polygon of support, is stable statically, and can tolerate large slopes without rollover. However, when this vehicle passes over bumps, dynamic disturbances at the wheels generate large torques, which tend to upset the vehicle about the roll, pitch, and yaw axes. In effect, the large polygon of support required for static stability provides a leverage mechanism for the generation of dynamic torque disturbances. Furthermore, the support points must comply with the surface, statically and dynamically, by control of support points (e.g., suspension) and/or vehicle attitude changes. Sophisticated vehicle suspensions have been developed to minimize dynamic disturbances, but passive-spring suspensions decrease static stability by allowing the center of mass to move toward the outside of the support polygon. Active suspensions may overcome this problem but require additional complexity and energy expenditures.

As a second example, consider a bicycle or motorcycle that has two wheels in the fore-aft (tandem) configuration. Such a vehicle is statically unstable in the roll direction but achieves dynamic stability at moderate speed through appropriate steering geometry and gyroscopic action of the steered front wheel. Steering stability generally increases with speed due to gyroscopic effects. Dynamic forces at the wheel-ground contact point act on or near the vehicle center (sagittal) plane and, thus, produce minimal roll disturbances. Additionally, the bicycle can remain upright when traveling on side slopes. Thus, sacrificing static roll stability enhances the dynamic roll stability and permits the vehicle to automatically adjust to side slopes.

As a logical extension of this argument, consider a wheel rolling down an incline. Under the influence of gravity, gyroscopic action causes the wheel to precess (the axis of wheel rotation turns) about the vertical axis—rather than simply falling sideways as it does when not rolling—and the wheel steers in the direction it is leaning. The resulting curved path of motion of the wheel on the ground produces radial (centrifugal) forces at the wheel-ground contact point, tending to right the wheel. Dynamic disturbances due to surface irregularities act through or near the wheel's center of mass, producing minimal torques in roll, pitch, and yaw. The angular momentum of the wheel, in addition to providing the natural gyroscopic steering mechanism, tends to stabilize the wheel with respect to roll and yaw. In terms of attitude control, the wheel is relatively insensitive to fore-aft and side slopes. The result is a highly stable rolling motion with minimal attitude disturbances and tolerance of fore-aft and vertical disturbances. One can readily observe this behavior by rolling an automobile tire down a bumpy hillside.

There are precedents for single-wheel-like vehicles. In 1869, R. C. Hemmings patented a velocipede, a large wheel encircling the rider that is powered by hand cranks. Palmer (1956) describes several single-wheel vehicles with an operator riding inside. *Modern Mechanix and Inventions* (1935) describes the gyroauto, which carried riders between a pair

of large, side-by-side wheels, and was claimed capable of a speed of 187 kph (116 mph). In the same publication is a description of the dyno-wheel, a concept having a buslike chassis straddling a huge central wheel. The relatively large diameter of a single-wheel vehicle enhances its obstacle-crossing ability, smoothness of motion, and rolling efficiency (Bekker, 1956). Furthermore, a single-track vehicle can more easily find obstacle-free paths on the ground, and its narrow profile can improve maneuverability. However, problems with steering, low-speed stability, and aerodynamics have kept such vehicles from becoming commonplace.

The purpose of this research is to exploit the natural steering behavior and stability of the rolling wheel in the development of a highly dynamic, single-wheel robot. We have built working models of such a vehicle and have demonstrated some of its potential capabilities.

3. Robot Concept

Gyrover is a novel, single-wheel, gyroscopically stabilized robot originally developed at Carnegie Mellon University (Brown and Xu, 1996). Figure 1 shows a schematic of the mechanism design. Essentially, Gyrover is a sharp-edged wheel with an actuation mechanism fitted inside the wheel. The actuation mechanism consists of three separate actuators: (1) a spin motor, which spins a suspended flywheel at a high rate, imparting dynamic stability to the robot; (2) a tilt motor, which controls the steering of Gyrover; and (3) a drive motor, which causes forward and/or backward acceleration by driving the single wheel directly.

The behavior of Gyrover is based on the principle of gyroscopic precession as exhibited in the stability of a rolling wheel. Because of its angular momentum, a spinning wheel tends to precess at right angles to an applied torque, according to the fundamental equation of gyroscopic precession:

$$T = J \times \omega \times \Omega \quad (1)$$

where ω is the angular speed of the wheel, Ω is the wheel's

precession rate normal to the spin axis, J is the wheel polar moment of inertia about the spin axis, and T is the applied torque normal to the spin and precession axes. Therefore, when a rolling wheel leans to one side, rather than just fall over, the gravitationally induced torque causes the wheel to precess so that it turns in the direction that it is leaning. Gyrover supplements this basic concept with the addition of an internal gyroscope—the spinning flywheel—nominally aligned with the wheel and spinning in the direction of forward motion. The flywheel's angular momentum produces lateral stability when the wheel is stopped or moving slowly.

Gyrover has a number of potential advantages over multi-wheeled vehicles:

1. The entire system can be enclosed within the wheel to provide mechanical and environmental protection for equipment and mechanisms.
2. Gyrover is resistant to getting stuck on obstacles because it has no body to hang up, no exposed appendages, and the entire exposed surface is live (driven).
3. The tiltable flywheel can be used to right the vehicle from its statically stable, rest position (on its side). The wheel has no backside on which to get stuck.
4. Gyrover can turn in place by simply leaning and precessing in the desired direction, with no special steering mechanism, which enhances maneuverability.
5. Single-point contact with the ground eliminates the need to accommodate uneven surfaces and simplifies control.
6. Full drive traction is available because all the weight is on the single drive wheel.
7. A large pneumatic tire may have very low ground-contact pressure, which results in minimal disturbance to the surface and minimum rolling resistance. The tire may be suitable for traveling on soft soils, sand, snow, or ice; brush or other vegetation; or, with adequate buoyancy, water.

Potential applications for Gyrover are numerous. Because it can travel on both land and water, it may find amphibious use on beaches or swampy areas for general transportation, exploration, rescue, or recreation. Similarly, with appropriate tread, it should travel well over soft snow with good traction and minimal rolling resistance. As a surveillance robot, Gyrover could use its slim profile to pass through doorways and narrow passages, and its ability to turn in place to maneuver in tight quarters. Another potential application is as a high-speed lunar vehicle, where the absence of aerodynamic disturbances and low gravity would permit efficient, high-speed mobility. As the development progresses, we anticipate that other, more specific uses will become evident.

4. Design and Implementation

We studied feasibility through basic analysis and simple experiments; we then designed and built two radio-controlled

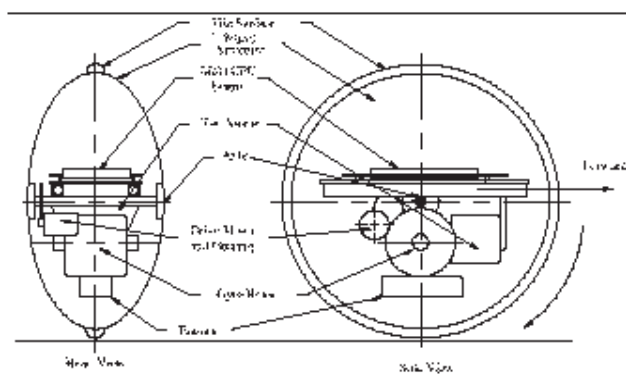


Fig. 1. The basic configuration of the robot.

(RC) working models. These have proven that the concept is workable and have verified many of the expected advantages.

Given a basic understanding of the gyroscopic principle and wheel dynamic stability, our first task was to find a mechanism for steering the wheel along a desired path. Turning (steering) of the wheel is the result of gyroscopic precession about the yaw axis, which is caused by roll torques as explained above. We considered two mechanisms for producing this torque: lateral shifting of weight within the vehicle and leaning of the wheel. With regard to the first approach, the need to provide adequate internal space for shifting large masses within the vehicle is a significant drawback. Moving the mass outside the wheel's envelope is unappealing because of the effective broadening of the vehicle and potential for the movable mass to contact the ground or other obstacles. For the entire wheel to lean, the complete weight of the wheel must shift laterally—not just a relatively small, movable mass—to generate the needed roll torque. Leaning of the wheel can be effected by the use of internal reaction wheels or masses, but these tend to acquire kinetic energy, become velocity saturated, and generate angular momentum that can corrupt vehicle behavior. Another way is to react against an internal gyroscope as described above; this mechanism has been implemented and operated successfully.

Several alternative configurations were considered. A spherical shape, which can be statically stable, does not exhibit the natural steering behavior resulting from the interaction of the gravitational (overturning) torque and the gyroscopic effect. In fact, a narrow tire-contact area is desirable for steering responsiveness (dependent on the gravitational torque for a given lean angle). Two wheels side by side provide static stability but are sensitive to roll disturbances, do not exhibit the same natural steering behavior, and will not roll stably on two wheels above a critical speed. (Observe the behavior of a short cylinder, such as a roll of tape, rolled along the floor.) Outboard wheels, either on the sides or front/back, were considered for static stability and steering effects, as well as acceleration and braking enhancement; but in addition to mechanical complexity, outboard wheels defeat the basic elegance and control simplicity of the concept. Actually, the robot has the potential to be statically stable to provide a solid base of support for sensors, instruments, small manipulators, and so on simply by resting on its side. The present concept, totally enclosed in the single wheel, provides a simple, reliable, rugged vehicle. Experimental work to date includes several simple experiments to verify the stability and steering principles of three working vehicles. The first vehicle, Gyrover I (Figure 2), was assembled from RC model airplane/car components and quickly confirmed the concept. The vehicle has a diameter of 29 cm and a mass of 2.0 kg. It can be easily driven and steered by remote control, has good high-speed stability on smooth or rough terrain, and can be kept standing in place. This vehicle traveled at over 10 kph, negotiated relatively rough terrain (a small gravel pile), and traversed a

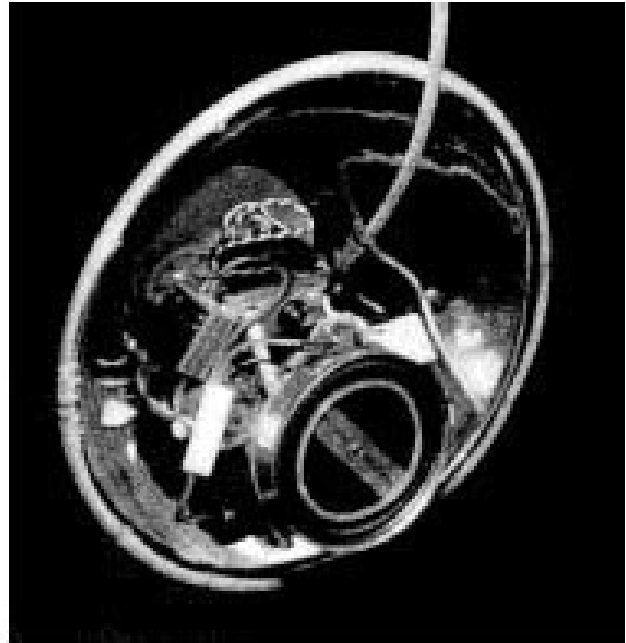


Fig. 2. The first prototype of the robot.

45° ramp 75% its height diameter. Recovery from falls (resting on the round side of the wheel) has been achieved with a strategy using both the wheel forward drive and gyro tilt control. The main shortcomings of this robot are its lack of resilience and vulnerability to wheel damage, excessive battery drain due to drag on the gyro (bearing and aerodynamic), inadequate torque in the tilt servo, and incomplete enclosure of the wheel.

The second vehicle, Gyrover II (Figure 2), was designed to address these problems. It is slightly larger than Gyrover I (34 cm diameter, 2.0 kg) and uses many RC model parts. Tilt-servo torque and travel were approximately doubled. Gyrover II uses a gyro housed in a vacuum chamber to cut power consumption by 80%, which increases battery life from about 10 min to 50 min. The entire robot is housed inside a specially designed pneumatic tire that protects the mechanism from mechanical and environmental abuse and provides an enclosure that is resilient, although less rugged than hoped. The robot contains a variety of sensors to monitor motor current, position and speed, tire and vacuum pressure, wheel/body orientation, and gyro temperature. Gyrover II has been assembled and driven by manual remote control on a smooth floor, and has demonstrated the ability to float and be controllable on water.

The third version, Gyrover III (Figure 4), was designed on a larger scale to permit it to carry numerous inertial sensors and a computer (486 PC) for data acquisition and/or control. This machine uses a lightweight 40-cm bicycle tire and rim and a pair of transparent domes attached to the axle. Overall weight is about 7 kg. Heavy duty RC motors and servo are

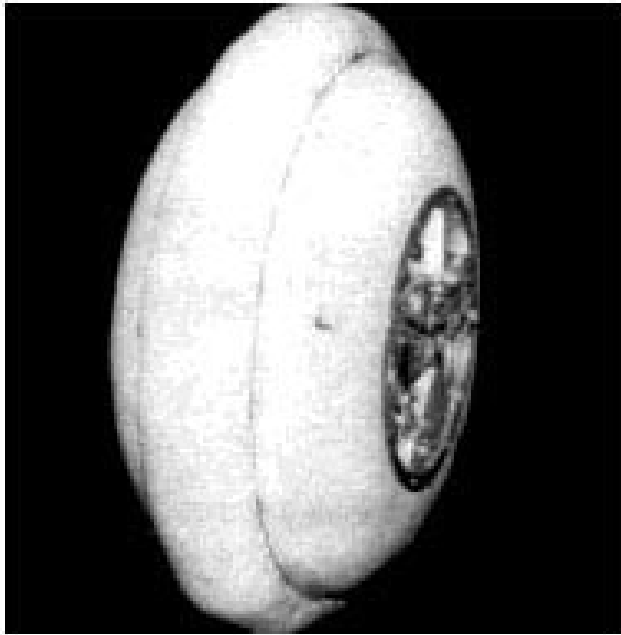


Fig. 3. The second prototype of the robot.

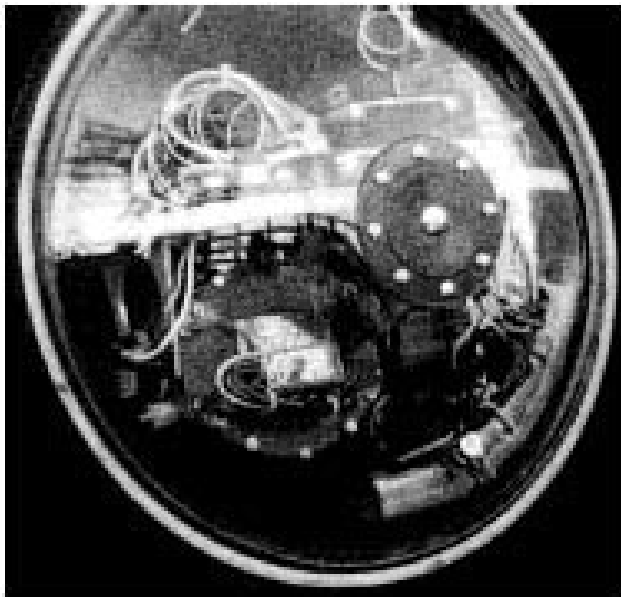


Fig. 4. The third prototype of the robot.

used to spin the gyro, drive the wheel forward, and control gyro tilt. Gyrover III travels up to 10 mph, recovers from falls, and runs about 25 min per charge of its NiCad batteries. It can carry a video camera that looks through the transparent wheel, and it can transmit video data to a remote receiver/monitor.

5. Kinematic Constraints: Holonomic and Non-holonomic

Thus far, Gyrover has been controlled only manually, using two joysticks to control the drive and tilt motors through a radio link. A complete dynamic model is necessary to develop automatic control of the system. In the following sections, we will develop the nonholonomic kinematic constraints and a dynamic model using the constrained generalized Lagrangian formulation.

5.1. Coordinate Frame

In deriving the equations of motion of the robot, we assume that the wheel is a rigid, homogeneous disk that rolls over a perfectly flat surface without slipping. We model the actuation mechanism, suspended from the wheel bearing, as a two-link manipulator, with a spinning disk attached at the end of the second link (Figure 5). The first link of length l_1 represents the vertical offset of the actuation mechanism from the axis of the Gyrover wheel. The second link of length l_2 represents the horizontal offset of the spinning flywheel and is relatively smaller compared to the vertical offset.

Next, we assign four coordinate frames as follows: (1) the inertial frame Σ_O , whose x - y -plane is anchored to the flat surface; (2) the body coordinate frame $\Sigma_B \{x_B, y_B, z_B\}$, whose origin is located at the center of the single wheel and whose z -axis represents the axis of rotation of the wheel; (3) the coordinate frame of internal mechanism $\Sigma_C \{x_c, y_c, z_c\}$, whose center is located at point D and whose z -axis is always

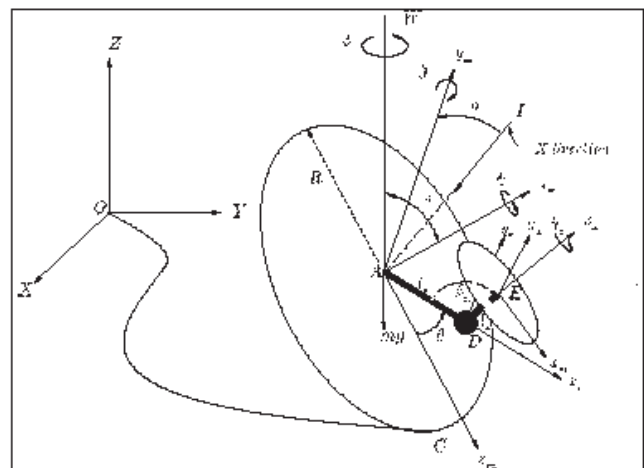


Fig. 5. Definition of coordinate frames and system variables.

Table 1. Variable Definitions

α, α_a	=	precession angles of the wheel and flywheel, respectively, measured about the vertical axis
β	=	lean angles of the wheel
β_a	=	tilt angle between the link l_1 and z_a -axis of the flywheel
γ, γ_a	=	spin angles of the wheel and flywheel, respectively
θ	=	angle between link l_1 and x_B -axis of the wheel
m_w, m_i, m_f	=	mass of the wheel, internal mechanism, and flywheel, respectively
m	=	total mass of the robot
R, r	=	radius of the wheel and flywheel, respectively
$I_{xxw}, I_{yyw}, I_{zzw}$	=	moment of inertia of the wheel about x , y , and z axes
$I_{xxf}, I_{yyf}, I_{zzf}$	=	moment of inertia of the flywheel about x , y , and z axes
μ_s, μ_g	=	friction coefficient in yaw and pitch directions, respectively
u_1, u_2	=	drive torque of the drive motor and tilt torque of the tilt motor, respectively

parallel to z_B ; and (4) the flywheel coordinate frame $\sum_E \{x_a, y_a, z_a\}$, whose center is located at the center of the Gyrover flywheel and whose z -axis represents the axis of rotation of the flywheel. Note that y_a is always parallel to y_c . The definitions and configurations of system and variables are shown in Table 1 and Figure 5. Rolling without slipping is a typical example of a nonholonomic system, since in most cases some of the constrained equations for the system are nonintegrable. Gyrover is a similar type of nonholonomic system. Here, we first derive the constraints of the single wheel. Then, we derive the dynamic model of Gyrover based on these constraints. We define (i, j, k) and (l, m, n) to be the unit vectors of the coordinate system $XYZO(\sum_O)$ and $x_B y_B z_B A(\sum_B)$, respectively. Let $S_x := \sin(x)$ and $C_x := \cos(x)$. The transformation between these two coordinate frames is given by

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = R_B^O \begin{bmatrix} l \\ m \\ n \end{bmatrix}, \quad (2)$$

where R_B^O is the rotation matrix from $\sum_O$ to $\sum_B$:

$$R_B^O = \begin{bmatrix} -S_\alpha C_\beta & -C_\alpha & -S_\alpha S_\beta \\ C_\alpha C_\beta & -S_\alpha & C_\alpha S_\beta \\ -S_\beta & 0 & C_\beta \end{bmatrix}. \quad (3)$$

Let v_A and ω_B denote the velocity of the center of mass of the single wheel and its angular velocity with respect to the inertia frame $\sum_O$. Then, we have

$$\omega_B = -\dot{\alpha} S_\beta l + \dot{\beta} m + (\dot{\gamma} + \dot{\alpha} C_\beta) n. \quad (4)$$

The constraints require that the disk rolls without slipping on the horizontal plane; that is, the velocity of the contact point on the disk is zero at any instant

$$v_c = 0, \quad (5)$$

where v_c is the velocity of contact point of the single wheel. Now, we can express v_A as

$$v_A = \omega_B \times r_{AC} + v_c, \quad (6)$$

where $r_{AC} = -Rl$ represents the vector from the frame C to A in Figure 5. Substituting eqs. (4) and (5) in eq. (6) gives

$$v_A = \dot{X}i + \dot{Y}j + \dot{Z}k, \quad (7)$$

where

$$\dot{X} = R(\dot{\gamma} C_\alpha + \dot{\alpha} C_\alpha C_\beta - \dot{\beta} S_\alpha S_\beta) \quad (8)$$

$$\dot{Y} = R(\dot{\gamma} S_\alpha + \dot{\alpha} C_\beta S_\alpha + \dot{\beta} C_\alpha S_\beta) \quad (9)$$

$$\dot{Z} = R\dot{\beta} C_\beta. \quad (10)$$

Equations (8) and (9) are nonintegrable and, hence, are nonholonomic, whereas eq. (10) is integrable; that is,

$$Z = RS_\beta. \quad (11)$$

Therefore, the robot can be represented by seven (e.g., $X, Y, \alpha, \beta, \gamma, \beta_a, \theta$) independent variables instead of eight.

6. Robot Dynamics

In this section, we study the equation of motion by calculating the Lagrangian $L = T - P$ of the system, where T and P are the kinetic energy and potential energy of the system, respectively. We divide the system into three parts: (1) single wheel, (2) internal mechanism, and (3) spinning flywheel.

6.1. Single Wheel

The kinetic energy of the single wheel is given by

$$T_w = \frac{1}{2} m_w [\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2] + \frac{1}{2} [I_{xxw} \omega_x^2 + I_{yyw} \omega_y^2 + I_{zzw} \omega_z^2]. \quad (12)$$

Substituting eqs.(4) and (10) in eq.(12) yields

$$T_w = \frac{1}{2} m_w [\dot{X}^2 + \dot{Y}^2 + (R\dot{\beta} C_\beta)^2] + \frac{1}{2} [I_{xxw} (\dot{\alpha} S_\beta)^2 + I_{yyw} \dot{\beta}^2 + I_{zzw} (\dot{\alpha} C_\beta + \dot{\gamma})^2]. \quad (13)$$

The potential energy of the single wheel is

$$P_w = m_w g R S_\beta. \quad (14)$$

6.2. Internal Mechanism and Spinning Flywheel

We need to compute the translational and rotational parts of kinetic energy for the internal mechanism and flywheel, respectively. We assume that l_2 is very small compared with l_1 ,

$$l_2 \simeq 0. \quad (15)$$

Thus, the flywheel's center of mass (E) coincides with that of the internal mechanism (D). Let $\{x_f, y_f, z_f\}$ be the center of mass of the internal mechanism and the flywheel w.r.t. $\sum_O$. The transformation from the center of mass of a single wheel to the flywheel can be described:

$$\begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + R_B^O \begin{bmatrix} l_1 C_\theta \\ l_1 S_\theta \\ 0 \end{bmatrix}. \quad (16)$$

Let T_f^t denote the translational kinetic energy of the flywheel and the internal mechanism:

$$T_f^t = \frac{1}{2}(m_i + m_f)[\dot{x}_f^2 + \dot{y}_f^2 + \dot{z}_f^2]. \quad (17)$$

Differentiating eq. (16) and substituting it in eq. (17), we obtain T_f^t . We observed that the internal mechanism swings slowly, so it should not contribute highly to the rotational kinetic energy. Let ω_f be the angular velocity of flywheel w.r.t. $\sum_O$. We then have

$$\omega_f = R_B^E \omega_B + \begin{bmatrix} 0 \\ \dot{\beta}_a \\ \dot{\gamma}_a \end{bmatrix}, \quad (18)$$

where R_B^E is the transformation from $\sum_B$ to $\sum_E$:

$$R_B^E = \begin{bmatrix} C_\theta S_{\beta_a} & -S_\theta S_{\beta_a} & C_{\beta_a} \\ S_\theta & C_\theta & 0 \\ C_\theta C_{\beta_a} & -C_{\beta_a} S_\theta & S_{\beta_a} \end{bmatrix}. \quad (19)$$

The rotational kinetic energy of the flywheel is now given by

$$T_f^r = \frac{1}{2} [(\omega_{fx})^2 I_{xxf} + (\omega_{fy})^2 I_{yyf} + (\omega_{fz})^2 I_{zzf}]. \quad (20)$$

The flywheel is assumed to be a uniform disk, and the principle moments of inertia are $I_{xxf} = I_{yyf} = \frac{1}{4} m_f r^2$, $I_{zzf} = \frac{1}{2} m_f r^2$. The potential energy of the flywheel and internal mechanism is

$$P_f = (m_i + m_f)(R S_\beta - l_1 C_\theta S_\beta)(g). \quad (21)$$

6.3. Lagrangian of the System

The Lagrangian of the system is

$$L = [T_w + (T_f^t + T_f^r)] - (P_w + P_f). \quad (22)$$

Substituting eqs. (12), (17), (20), (14), and (21) in eq. (22), we may determine L . There are only two control torques available on the system. One is drive torque (u_1), and the other is tilt torque (u_2). Consequently, using the constrained Lagrangian method, the dynamic equation of the entire system is given by

$$M(q)\ddot{q} + N(q, \dot{q}) = A^T \lambda + Bu, \quad (23)$$

where $M(q) \in R^{7 \times 7}$ and $N(q, \dot{q}) \in R^{7 \times 1}$ are the inertia matrix and nonlinear terms, respectively:

$$A(q) = \begin{bmatrix} 1 & 0 & -RC_\alpha C_\beta & RS_\alpha S_\beta & -RC_\alpha & 0 & 0 \\ 0 & 1 & -RC_\beta S_\alpha & -RC_\alpha S_\beta & -RS_\alpha & 0 & 0 \end{bmatrix} \quad (24)$$

$$q = \begin{bmatrix} X \\ Y \\ \alpha \\ \beta \\ \gamma \\ \beta_a \\ \theta \end{bmatrix} \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_1 & 0 \\ 0 & 1 \\ k_2 & 0 \end{bmatrix} u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

The nonholonomic constraints can be written as

$$A(q)\dot{q} = 0. \quad (25)$$

Note that all elements of the last two columns of the matrix A are zero, since the nonholonomic constraints restrict the motion of the single wheel, not the flywheel. The last two columns represent the motion variables of the flywheel. Moreover, the matrix B has only three rows with nonzero elements, since the input torques drive only the tilt angle of the flywheel (β_a) and the rotating angle of the single wheel (γ). The fifth and sixth rows of B are nonzero, since they represent the tilting motion of the flywheel and the rotating motion of the single wheel, respectively. Furthermore, when the single wheel rotates, the pendulum motion of internal mechanism is introduced; thus, θ changes. Therefore, the drive torque of the single wheel will also affect the pendulum motion of the internal mechanism (θ), so that the seventh row of the matrix B is not zero.

6.4. Normal Form of the System

In this section, we will eliminate the Lagrange multipliers so that a minimum set of differential equations is obtained in a manner similar to Bloch, Reyhanoglu, and McClamroch (1992). We first partition the matrix $A(q)$ into A_1 and A_2 , where $A = [A_1 : A_2]$:

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A_2 = \begin{bmatrix} -RC_\alpha C_\beta & RS_\alpha S_\beta & -RC_\alpha & 0 & 0 \\ -RC_\beta S_\alpha & -RC_\alpha S_\beta & -RS_\alpha & 0 & 0 \end{bmatrix}.$$

Let

$$C(q) = \begin{bmatrix} -A_1^{-1}A_2 \\ I_{3 \times 3} \end{bmatrix}. \quad (26)$$

Then, consider the following relationship:

$$\dot{q} = C(q)\dot{q}_2, \quad (27)$$

where $q_1 = [X, Y]^T$ and $q_2 = [\alpha, \beta, \gamma, \beta_a, \theta]^T$. Differentiating eq. (27) yields

$$\ddot{q} = C(q)\ddot{q}_2 + \dot{C}(q)\dot{q}_2. \quad (28)$$

Substituting eq. (28) into eq. (23) and premultiplying both sides by $C^T(q)$ gives

$$\begin{aligned} & C^T(q)M(q)C(q)\ddot{q}_2 \\ &= C^T(q)[Bu - N(q, C(q)\dot{q}_2) - M(q)\dot{C}(q)\dot{q}_2], \end{aligned} \quad (29)$$

where $C^T(q)M(q)C(q)$ is a 5×5 symmetric positive definite matrix function. Note that eq.(29) depends only on $(\alpha, \beta, \gamma, \beta_a, \theta)$. By the numerical integration, we can obtain q_2 from $\ddot{q}_2$ in eq. (29), and then obtain (X, Y) by substituting q_2 and $\dot{q}_2$ in eq. (27).

7. Characteristics of Robot Dynamics

Understanding the characteristics of the robot dynamics is significant in the control of the system. To this end, we first simplify the model. Practically, we may assume $I_{xxw} = I_{yyw} = \frac{1}{2}m_w R^2$, $I_{zzw} = m_w R^2$, and l_1 and l_2 are zero; thus, the mass center of flywheel is coincident with the center of the robot. For steady motion of the robot, the pendulum motion of the internal mechanism is sufficiently small to be neglected; thus, θ is set to be zero. The spinning rate of the flywheel γ_a is set to be constant. Let $S_{\beta, \beta_a} := \sin(\beta + \beta_a)$, $C_{\beta, \beta_a} := \cos(\beta + \beta_a)$, and $S_{2\beta, \beta_a} := \sin[2(\beta + \beta_a)]$. Based on the previous derivation, the normal form of the dynamics model is

$$M(q)\ddot{q} = F(q, \dot{q}) + Bu \quad (30)$$

$$\dot{X} = R(\dot{\gamma}C_\alpha + \dot{\alpha}C_\alpha C_\beta - \dot{\beta}S_\alpha S_\beta) \quad (31)$$

$$\dot{Y} = R(\dot{\gamma}S_\alpha + \dot{\alpha}C_\beta S_\alpha + \dot{\beta}C_\alpha S_\beta), \quad (32)$$

where $q = [\alpha, \beta, \gamma, \beta_a]^T$,

$$M = \begin{bmatrix} M_{11} & 0 & M_{13} & 0 \\ 0 & I_{xxf} + I_{xxw} + mR^2 & 0 & I_{xxf} \\ M_{13} & 0 & 2I_{xxw} + mR^2 & 0 \\ 0 & I_{xxf} & 0 & I_{xxf} \end{bmatrix}$$

$$F = [F_1, F_2, F_3, F_4]^T$$

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$M_{11} = I_{xxf} + I_{xxw} + I_{xxw}C_\beta^2 + mR^2C_\beta^2 + I_{xxf}C_{\beta, \beta_a}^2$$

$$M_{13} = 2I_{xxw}C_\beta + mR^2C_\beta$$

$$\begin{aligned} F_1 &= (I_{xxw} + mR^2)S_{2\beta}\dot{\alpha}\dot{\beta} + I_{xxf}S_{2\beta\beta_a}\dot{\alpha}\dot{\beta} \\ &+ I_{xxf}S_{2\beta\beta_a}\dot{\alpha}\dot{\beta}_a + 2I_{xxw}S_\beta\dot{\beta}\dot{\gamma} + 2I_{xxf}S_{\beta, \beta_a}\dot{\beta}\dot{\gamma}_a \\ &+ 2I_{xf}S_{\beta, \beta_a}\dot{\beta}_a\dot{\gamma}_a - \mu_s\dot{\alpha} \end{aligned}$$

$$\begin{aligned} F_2 &= -gmRC_\beta - (I_{xxw} + mR^2)C_\beta S_\beta\dot{\alpha}^2 \\ &- I_{xxf}C_{\beta, \beta_a}S_{\beta, \beta_a}\dot{\alpha}^2 - (2I_{xxw} + mR^2)S_\beta\dot{\alpha}\dot{\gamma} \\ &- 2I_{xxf}S_{\beta, \beta_a}\dot{\alpha}\dot{\gamma}_a \end{aligned}$$

$$F_3 = 2(I_{xxw} + mR^2)S_\beta\dot{\alpha}\dot{\beta}$$

$$F_4 = -I_{xxf}C_{\beta, \beta_a}S_{\beta, \beta_a}\dot{\alpha}^2 - 2I_{xxf}S_{\beta, \beta_a}\dot{\alpha}\dot{\gamma}_a,$$

where (X, Y, Z) are the coordinates of center of mass of the single wheel with respect to the inertial frame as shown in Figure 5. $M(q) \in R^{4 \times 4}$ and $N(q, \dot{q}) \in R^{4 \times 1}$ are the inertial matrix and nonlinear terms, respectively. Equations (30), (31), and (32) form the dynamics model and nonholonomic velocity constraints of the robot.

We further simplify the model by decoupling the tilting variable β_a from eq. (30). Practically, β_a is directly controlled by the tilt motor (position control), assuming that the tilt actuator has an adequate torque to track the desired $\beta_a(t)$ trajectory exactly. Therefore, β_a can be decoupled from eq. (30). This is similar to the case of decoupling the steering variable from the bicycle dynamics shown in Beznos et al. (1998) and Getz (1994).

Since we consider $\dot{\beta}_a$ as a new input u_{β_a} , the dynamics model eq. (30) becomes

$$\begin{aligned} \dot{\beta}_a &= u_{\beta_a} \\ \tilde{M}(\tilde{q})\ddot{\tilde{q}} &= \tilde{F}(\tilde{q}, \dot{\tilde{q}}) + \tilde{B}\tilde{u}, \end{aligned} \quad (33)$$

with $\tilde{q} = [\alpha, \beta, \gamma]^T$,

$$\tilde{M} = \begin{bmatrix} M_{11} & 0 & M_{13} \\ 0 & I_{xxf} + I_{xxw} + mR^2 & 0 \\ M_{13} & 0 & 2I_{xxw} + mR^2 \end{bmatrix}$$

$$\tilde{F} = [\tilde{F}_1, \tilde{F}_2, \tilde{F}_3]^T$$

$$\tilde{B} = \begin{bmatrix} 0 & 0 & 1 \\ \tilde{B}_{12} & 0 & 0 \end{bmatrix}^T \tilde{u} = \begin{bmatrix} u_1 \\ u_{\beta_a} \end{bmatrix}$$

$$\begin{aligned} \tilde{F}_1 &= (I_{xxw} + mR^2)S_{2\beta}\dot{\alpha}\dot{\beta} + I_{xxf}S_{2\beta\beta_a}\dot{\alpha}\dot{\beta} \\ &- 2I_{xxw}S_\beta\dot{\beta}\dot{\gamma} + 2I_{xxf}S_{\beta, \beta_a}\dot{\beta}\dot{\gamma}_a - \mu_s\dot{\alpha} \end{aligned}$$

$$\tilde{F}_2 = F_2$$

$$\tilde{F}_3 = F_3$$

$$\tilde{B}_{12} = I_{xxf}S_{2\beta\beta_a}\dot{\alpha} + 2I_{xxf}S_{\beta, \beta_a}\dot{\gamma}_a.$$

Equation (33) shows the reduced dynamic model of the single-wheel robot after decoupling the tilting variable β_a , with new matrices $\tilde{M}(\tilde{q}) \in R^{3 \times 3}$ and $\tilde{F}(\tilde{q}, \dot{\tilde{q}}) \in R^{3 \times 1}$. The realistic

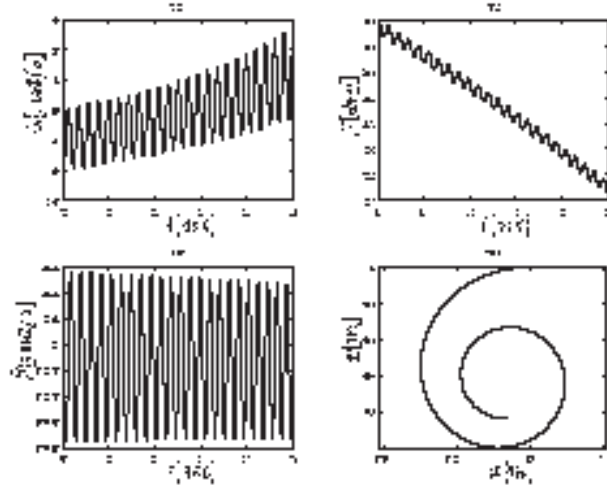


Fig. 6. The simulation results of a rolling disk without flywheel.

geometric/mass parameters are shown in Table 2. Note that if the lean angle β is set to be 0° or 180° , the single-wheel robot's inertial matrix $\tilde{M}$ will become singular because it violates the assumption of rolling without slipping, and the robot falls to the ground.

The robot dynamic model is highly coupled, nonholonomic, and underactuated. Because the major part of the robot is a rolling wheel, it precesses in a manner typical of a rolling disk. A rolling disk does not fall when it is rolling because a gyroscopic torque, resulting from the coupling motion between the roll and yaw motions, balances the gravitational torque. However, its rolling rate must be high enough to provide a sufficient gyroscopic torque for maintaining its balance. When we installed a flywheel on the wheel, the robot's gyroscopic torque was greater and depended less on its rolling speed $\dot{\gamma}$, owing to the high-spinning flywheel. That is, the lean angle of the robot tends to remain unchanged. We can explain this characteristic based on the equilibrium solution of eq. (33). For the low yaw rate and $\ddot{\alpha} = \ddot{\beta} = \ddot{\gamma} = 0$, $\dot{\beta} = 0$, $u_1 = u_{\beta_a} = 0$, eq. (33) becomes

$$0 = gmRC_{\beta} + (2I_{xxw} + mR^2)S_{\beta}\dot{\alpha}\dot{\gamma} + 2I_{xxf}S_{\beta,\beta_a}\dot{\alpha}\dot{\gamma}_a. \quad (34)$$

From eq. (34), the terms $\dot{\gamma}\dot{\alpha}$ and $\dot{\gamma}_a\dot{\alpha}$ are used to cancel the gravitational torques $gmRC_{\beta}$ in order to stabilize the roll component of the system. Because the spinning rate $\dot{\gamma}_a$ is very high, the term $2I_{xxf}S_{\beta,\beta_a}\dot{\alpha}\dot{\gamma}_a$ is significantly large in order to cancel the gravitational torque, even though the rolling speed $\dot{\gamma}$ is low. Thus, the system will achieve an equilibrium steering rate $\dot{\alpha}_s$,

$$\dot{\alpha}_s = \frac{-gmRC_{\beta_s}}{2I_{xxf}S_{\beta_s,\beta_a}\dot{\gamma}_a + (2I_{xxw} + mR^2)S_{\beta_s}\dot{\gamma}}, \quad (35)$$

for a specific lean angle β_s .

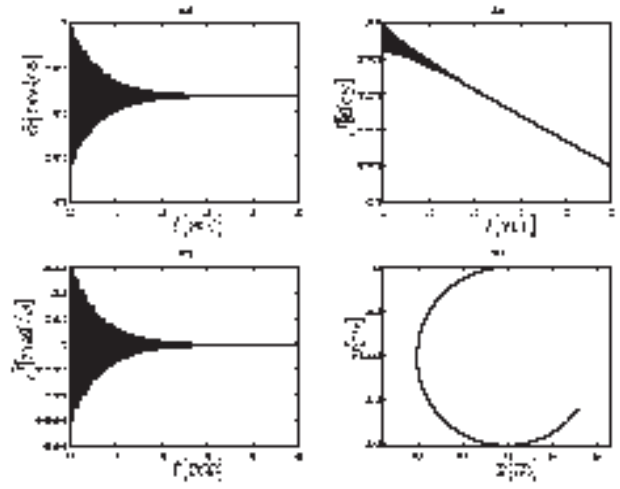


Fig. 7. The simulation results of the single-wheel robot.

Figures 6 and 7 show the simulation results of a rolling disk without the flywheel and the single-wheel robot, respectively, under the same initial conditions:

$$\begin{cases} \beta = 70^\circ, \beta_a = 0^\circ, \\ \dot{\beta} = \dot{\alpha} = \dot{\beta}_a = 0 \text{ rad/s}, \dot{\gamma} = 15 \text{ rad/s}, \\ \alpha = 0^\circ. \end{cases}$$

Note that the lean angle β of a rolling disk without the flywheel decreases much more rapidly than that of the single-wheel robot as shown in Figures 6b and 7b. This verifies the stabilizing effect of the flywheel on the single-wheel robot. In Figures 7a-c, under the influence of friction in the yaw direction, the steering rate $\dot{\alpha}$ and the leaning rate $\dot{\beta}$ will converge to a steady-state solution as shown in eq. (35). Otherwise, the unwanted high-frequency oscillation will occur. Practically, it will not produce a significant effect on the system because the frequency of the above oscillation is much greater than the system response. If the rolling rate is reduced, the rolling disk will fall much more quickly than it did in the previous case.

Until now, we consider only the case in which the flywheel's orientation is fixed with respect to the single wheel. Here, we will focus on the tilting effect of the flywheel on the robot. Based on the conservation of angular momentum, when there is a change in the tilt angle of the flywheel β_a , the whole robot will rotate in the opposite direction in order to maintain constant angular momentum. This implies that we may control the lean angle of the robot for steering. Simulation and experimental results are shown in Figures 8 and 9, respectively, under the same initial conditions:

$$\begin{cases} \beta = 90^\circ, \beta_a = \alpha = 0^\circ, \\ \dot{\beta} = \dot{\alpha} = \dot{\beta}_a = 0 \text{ rad/s}, \dot{\gamma} = 15 \text{ rad/s}, \\ \alpha = 0^\circ. \end{cases}$$

Table 2. Parameters Used in Simulation and Experiments

Single-wheel parameters:	$m = 1.25$ kg, $R = 17$ cm
Internal mechanism :	$m_i = 4.4$ kg
Flywheel parameters :	$m_f = 2.4$ kg, $r = 5$ cm
Friction coefficients :	$\mu_s = 1$ Nm/(rad/s), $\mu_g = 0.01$ Nm/(rad/s)

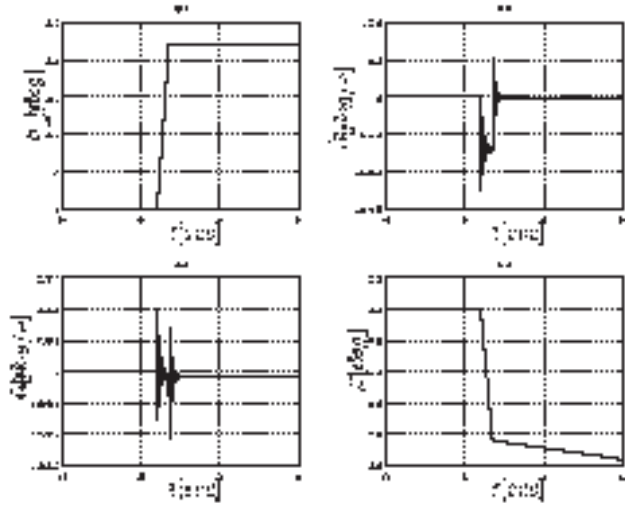


Fig. 8. The simulation results of tilting the flywheel of the robot with $\dot{\beta}_a = 73^\circ/\text{s}$.

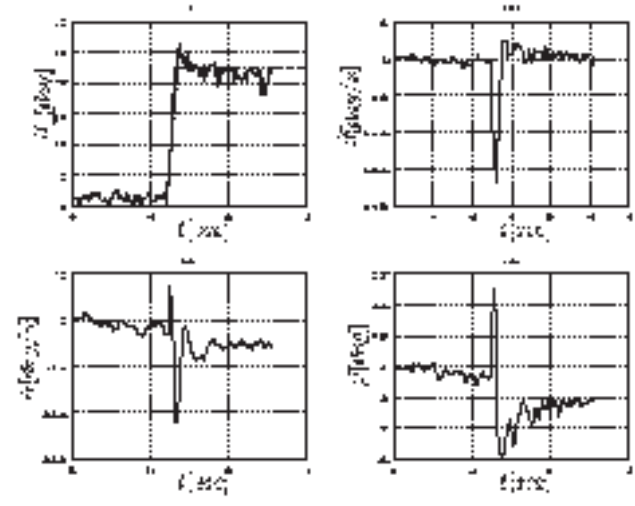


Fig. 9. The experimental results of tilting the flywheel of the robot with $\dot{\beta}_a = 73^\circ/\text{s}$.

Figures 8 and 9 show that if the tilt angle β_a rotates in $73^\circ/\text{s}$ counterclockwise at $t = 2.4$ s, the lean angle β rotates in a clockwise direction. In the experiment, the transient response of β is more critical than the simulations. For 2.7 s, the tilt angle remains unchanged, and β and the steering rate $\dot{\alpha}$ converge to a steady position in both simulations and experiments. In the experiment, we found no high-frequency oscillations, perhaps because the sensor response and the sampling time are much slower than the high-frequency oscillations.

8. Conclusion

In this paper, we proposed a novel concept of mobility that provides dynamic stability and rapid locomotion capability. We presented the mechanism design and three prototypes of a single-wheel robot. We then studied the velocity constraints and developed a dynamic model using the constrained Lagrangian method. The simulation study validated the robot concept and basic properties of the system. The robot is dynamically stable for locomotion on rough terrain, and its design offers significant advantages over multiwheel, statically stable vehicles. The robot concept provides a rich array of sensing, control, and planning issues to study. We are currently working on these issues and developing automatic control and planning schemes for a variety of applications.

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